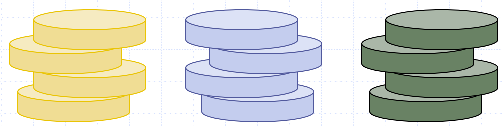
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

Stacks

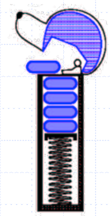


Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element



- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Assumes null is returned from top() and pop() when stack is empty
- Different from the built-in Java class java.util.Stack

```
public interface Stack<E> {
  int size();
  boolean isEmpty();
  E top();
  void push(E element);
  E pop();
}
```


Example

Method	Return Value	Stack Contents
push(5)	–	(5)
push(3)	–	(5, 3)
size()	2	(5, 3)
pop()	3	(5)
isEmpty()	false	(5)
pop()	5	()
isEmpty()	true	()
pop()	null	()
push(7)	–	(7)
push(9)	–	(7, 9)
top()	9	(7, 9)
push(4)	–	(7, 9, 4)
size()	3	(7, 9, 4)
pop()	4	(7, 9)
push(6)	–	(7, 9, 6)
push(8)	–	(7, 9, 6, 8)
pop()	8	(7, 9, 6)

Exceptions vs. Returning Null

- Attempting the execution of an operation of an ADT may sometimes cause an error condition
- Java supports a general abstraction for errors, called exception
- An exception is said to be "thrown" by an operation that cannot be properly executed
- In our Stack ADT, we do not use exceptions
- Instead, we allow operations pop and top to be performed even if the stack is empty
- For an empty stack, pop and top simply return null

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

```
main() {
  int i = 5;
  foo(i);
  }
foo(int j) {
  int k;
  k = j+1;
  bar(k);
  }
bar(int m) {
  …
  }
```

bar
PC = 1
m = 6

foo
PC = 3
j = 5
k = 6

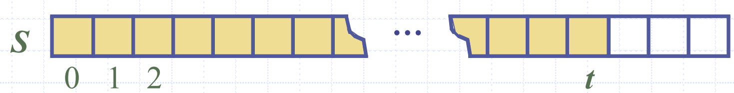
main
PC = 2
i = 5

Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

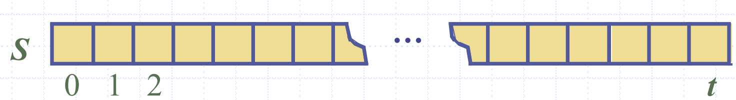
Algorithm *pop*()
  if *isEmpty*() **then**
    return null
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm *push*(*o*)
  if $t = S.length - 1$ **then**
    throw *IllegalStateException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Array-based Stack in Java

```
public class ArrayStack<E>
      implements Stack<E> {

   // holds the stack elements
   private E[ ] S;

   // index to top element
   private int top = -1;

   // constructor
   public ArrayStack(int capacity) {
       S = (E[ ]) new Object[capacity]);
   }
```

```
public E pop() {
  if isEmpty()
        return null;
   E temp = S[top];
   // facilitate garbage collection:
   S[top] = null;
   top = top – 1;
   return temp;
 }
… (other methods of Stack interface)
```

Example Use in Java

```
public class Tester {
    // … other methods
    public intReverse(Integer a[]) {
        Stack<Integer> s;
                    s = new
ArrayStack<Integer>();
        … (code to reverse array a) …
    }
```

```
    public floatReverse(Float f[]) {
        Stack<Float> s;
        s = new ArrayStack<Float>();
        … (code to reverse array f) …
    }
```


Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parenthesis Matching (Java)

```java
public static boolean isMatched(String expression) {
  final String opening = "({["; // opening delimiters
  final String closing = ")}]"; // respective closing delimiters
  Stack<Character> buffer = new LinkedStack<>( );
  for (char c : expression.toCharArray( )) {
    if (opening.indexOf(c) != −1) // this is a left delimiter
      buffer.push(c);
    else if (closing.indexOf(c) != −1) { // this is a right delimiter
      if (buffer.isEmpty( )) // nothing to match with
        return false;
      if (closing.indexOf(c) != opening.indexOf(buffer.pop( )))
        return false; // mismatched delimiter
    }
  }
  return buffer.isEmpty( ); // were all opening delimiters matched?
}
```


HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```html
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

HTML Tag Matching (Java)

```java
public static boolean isHTMLMatched(String html) {
  Stack<String> buffer =  new LinkedStack<>( );
  int j = html.indexOf('<'); // find first '<' character (if any)
  while (j != −1) {
    int k = html.indexOf('>', j+1); // find next '>' character
    if (k == −1)
      return false; // invalid tag
    String tag = html.substring(j+1, k); // strip away < >
    if (!tag.startsWith("/")) // this is an opening tag
      buffer.push(tag);
    else { // this is a closing tag
      if (buffer.isEmpty( ))
        return false; // no tag to match
      if (!tag.substring(1).equals(buffer.pop( )))
        return false; // mismatched tag
    }
    j = html.indexOf('<', k+1); // find next '<' character (if any)
  }
  return buffer.isEmpty( ); // were all opening tags matched?
}
```


Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group
evaluated from left to right
Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top())
    doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

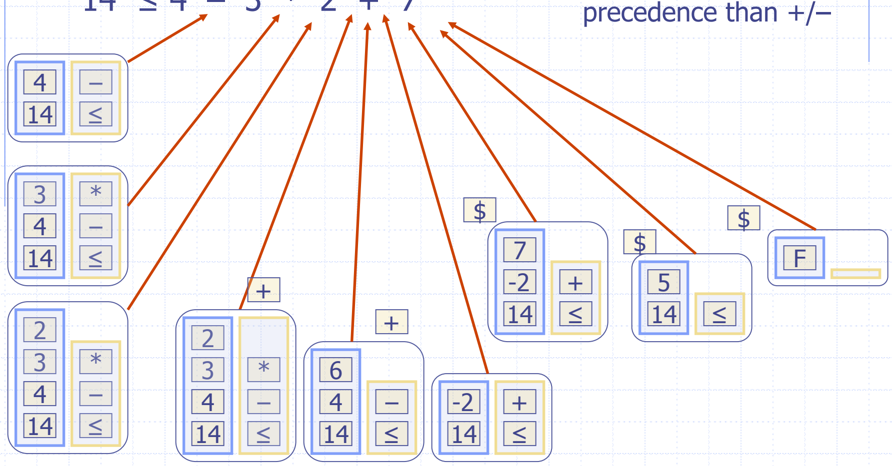
```
while there's another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```


Algorithm on an Example Expression

Slide by Matt Stallmann included with permission.

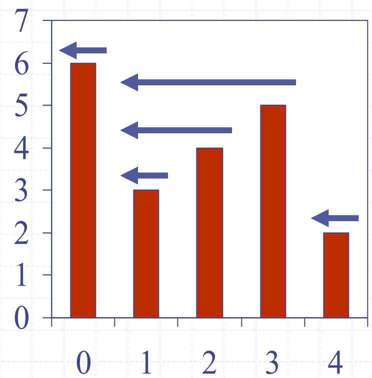
14 ≤ 4 – 3 * 2 + 7

Operator ≤ has lower precedence than +/–



Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \le X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

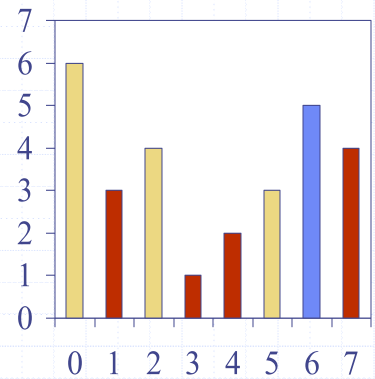
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	#
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \le i \wedge X[i - s] \le X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm *spans1* runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push x onto the stack



Linear Time Algorithm

- Each index of the array
  - Is pushed into the stack exactly one
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm *spans2* runs in $O(n)$ time

```
Algorithm spans2(X, n)                        #
    S ← new array of n integers               n
    A ← new empty stack                       1
    for i ← 0 to n − 1 do                     n
        while (¬A.isEmpty() ∧
                X[A.top()] ≤ X[i] ) do        n
            A.pop()                           n
        if A.isEmpty() then                   n
            S[i] ← i + 1                      n
        else
            S[i] ← i − A.top()                n
        A.push(i)                             n
    return S                                  1
```