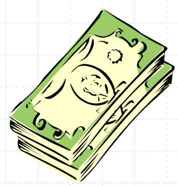
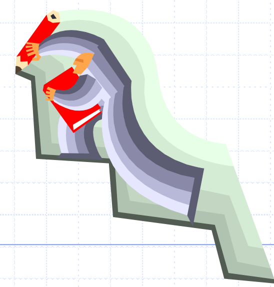
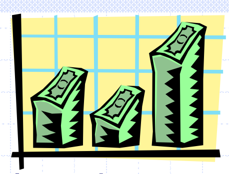
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

The Greedy Method and Text Compression





The Greedy Method Technique



- **The greedy method** is a general algorithm design paradigm, built on the following elements:
  - **configurations**: different choices, collections, or values to find
  - **objective function**: a score assigned to configurations, which we want to either maximize or minimize
- It works best when applied to problems with the **greedy-choice** property:
  - a globally-optimal solution can always be found by a series of local improvements from a starting configuration.

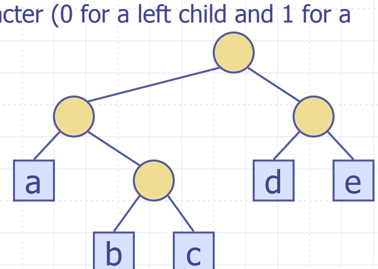
Text Compression

- Given a string X, efficiently encode X into a smaller string Y
  - Saves memory and/or bandwidth
- A good approach: **Huffman encoding**
  - Compute frequency f(c) for each character c.
  - Encode high-frequency characters with short code words
  - No code word is a prefix for another code
  - Use an optimal encoding tree to determine the code words

Encoding Tree Example

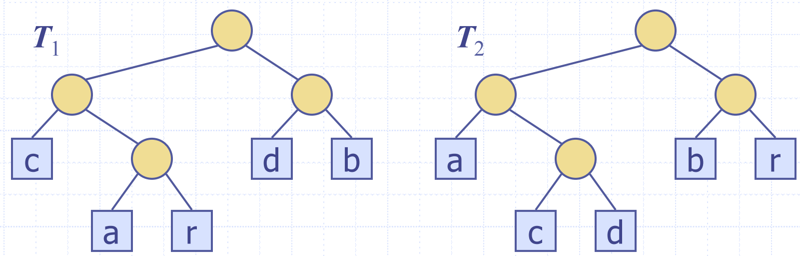
- A **code** is a mapping of each character of an alphabet to a binary code-word
- A **prefix code** is a binary code such that no code-word is the prefix of another code-word
- An **encoding tree** represents a prefix code
  - Each external node stores a character
  - The code word of a character is given by the path from the root to the external node storing the character (0 for a left child and 1 for a right child)

00	010	011	10	11
a	b	c	d	e



Encoding Tree Optimization

- Given a text string X, we want to find a prefix code for the characters of X that yields a small encoding for X
  - Frequent characters should have long code-words
  - Rare characters should have short code-words
- Example
  - X = abracadabra
  - T_1 encodes X into 29 bits
  - T_2 encodes X into 24 bits



Huffman's Algorithm

- Given a string X, Huffman's algorithm construct a prefix code the minimizes the size of the encoding of X
- It runs in time $O(n + d \log d)$, where n is the size of X and d is the number of distinct characters of X
- A heap-based priority queue is used as an auxiliary structure

Huffman's Algorithm

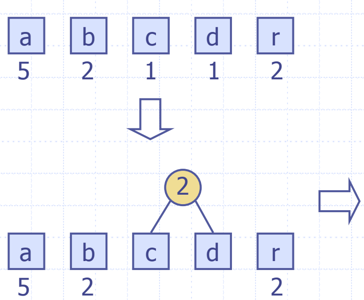
Algorithm Huffman(X):
  Input: String X of length n with d distinct characters
  Output: Coding tree for X

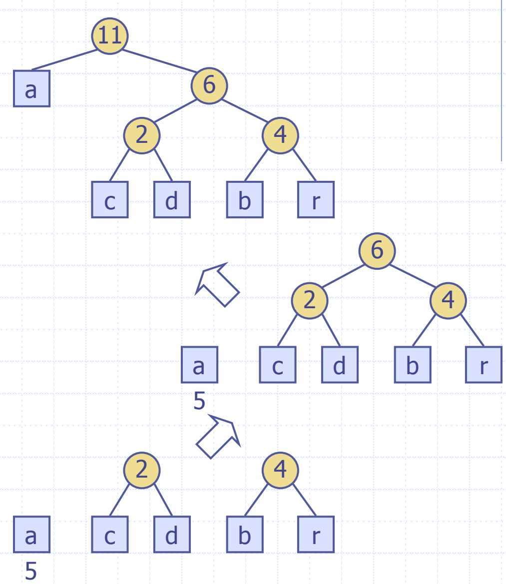
```
Compute the frequency f(c) of each character c of X.
Initialize a priority queue Q.
for each character c in X do
    Create a single-node binary tree T storing c.
    Insert T into Q with key f(c).
while len(Q) > 1 do
    (f1, T1) = Q.remove_min()
    (f2, T2) = Q.remove_min()
    Create a new binary tree T with left subtree T1 and right subtree T2.
    Insert T into Q with key f1 + f2.
(f, T) = Q.remove_min()
return tree T
```


Example

X = abracadabra
Frequencies

a	b	c	d	r
5	2	1	1	2

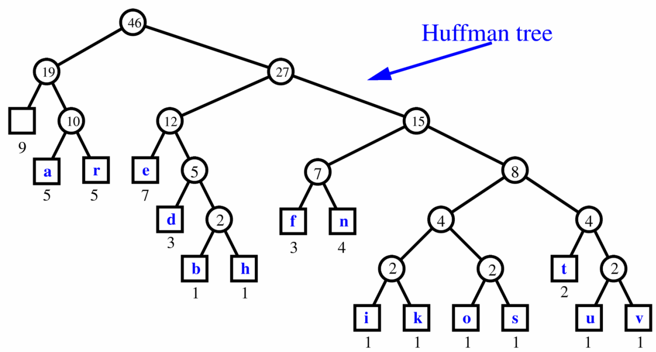




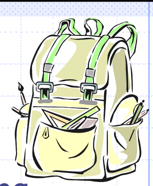
Extended Huffman Tree Example

String: **a fast runner need never be afraid of the dark**

Character		a	b	d	e	f	h	i	k	n	o	r	s	t	u	v
Frequency	9	5	1	3	7	3	1	1	1	4	1	5	1	2	1	1

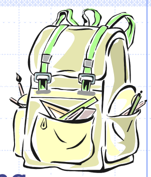


The Fractional Knapsack Problem (not in book)

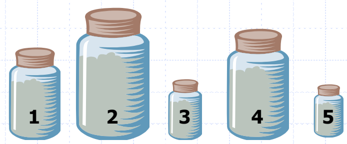


- Given: A set S of n items, with each item i having
  - b_i - a positive benefit
  - w_i - a positive weight
- Goal: Choose items with maximum total benefit but with weight at most W.
- If we are allowed to take fractional amounts, then this is the **fractional knapsack problem**.
  - In this case, we let x_i denote the amount we take of item i
  - Objective: maximize $\sum_{i \in S} b_i (x_i / w_i)$
  - Constraint: $\sum_{i \in S} x_i \le W$

Example



- Given: A set S of n items, with each item i having
  - b_i - a positive benefit
  - w_i - a positive weight
- Goal: Choose items with maximum total benefit but with weight at most W.



Items:	1	2	3	4	5
Weight:	4 ml	8 ml	2 ml	6 ml	1 ml
Benefit:	\$12	\$32	\$40	\$30	\$50
Value: (\$ per ml)	3	4	20	5	50

"knapsack"

10 ml

Solution:
- 1 ml of 5
- 2 ml of 3
- 6 ml of 4
- 1 ml of 2

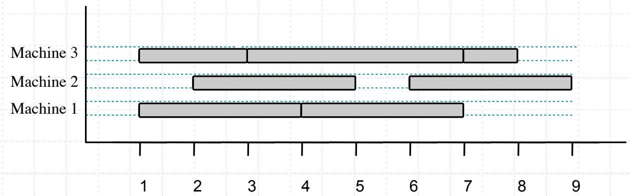
The Fractional Knapsack Algorithm

- Greedy choice: Keep taking item with highest value (benefit to weight ratio)
  - Since $\sum_{i \in S} b_i (x_i / w_i) = \sum_{i \in S} (b_i / w_i) x_i$
  - Run time: O(n log n). Why?
- Correctness: Suppose there is a better solution
  - there is an item i with higher value than a chosen item j, but $x_i<w_i$, $x_j>0$ and $v_i<v_j$
  - If we substitute some i with j, we get a better solution
  - How much of i: $\min\{w_i-x_i, x_j\}$
  - Thus, there is no better solution than the greedy one

Algorithm *fractionalKnapsack*(*S*, *W*)
  Input: set S of items w/ benefit b_i and weight w_i; max. weight W
  Output: amount x_i of each item i to maximize benefit w/ weight at most W
  for *each item i in S*
    $x_i \leftarrow 0$
    $v_i \leftarrow b_i / w_i$ {value}
  $w \leftarrow 0$ {total weight}
  while $w < W$
    remove item i w/ highest v_i
    $x_i \leftarrow \min\{w_i, W - w\}$
    $w \leftarrow w + \min\{w_i, W - w\}$

Task Scheduling (not in book)

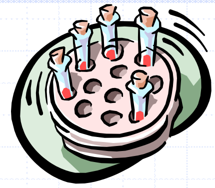
- Given: a set T of n tasks, each having:
  - A start time, s_i
  - A finish time, f_i (where $s_i < f_i$)
- Goal: Perform all the tasks using a minimum number of "machines."



Task Scheduling Algorithm

- Greedy choice: consider tasks by their start time and use as few machines as possible with this order.
  - Run time: O(n log n). Why?
- Correctness: Suppose there is a better schedule.
  - We can use k-1 machines
  - The algorithm uses k
  - Let i be first task scheduled on machine k
  - Machine i must conflict with k-1 other tasks
  - But that means there is no non-conflicting schedule using k-1 machines

Algorithm ***taskSchedule(T)***
  Input: set T of tasks w/ start time s_i and finish time f_i
  Output: non-conflicting schedule with minimum number of machines
  $m \leftarrow 0$ {no. of machines}
  while *T is not empty*
    remove task i w/ smallest s_i
    if *there's a machine j for i* **then**
      schedule i on machine j
    else
      $m \leftarrow m + 1$
      schedule i on machine m



Example

- Given: a set T of n tasks, each having:
  - A start time, s_i
  - A finish time, f_i (where $s_i < f_i$)
  - [1,4], [1,3], [2,5], [3,7], [4,7], [6,9], [7,8] (ordered by start)
- Goal: Perform all tasks on min. number of machines