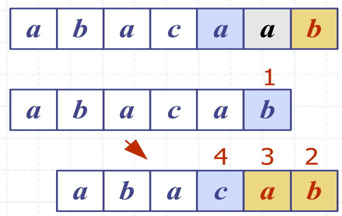
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

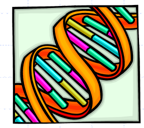
Pattern Matching



Strings

- A string is a sequence of characters
- Examples of strings:
  - Python program
  - HTML document
  - DNA sequence
  - Digitized image
- An alphabet Σ is the set of possible characters for a family of strings
- Example of alphabets:
  - ASCII
  - Unicode
  - {0, 1}
  - {A, C, G, T}
- Let P be a string of size m
  - A substring $P[i..j]$ of P is the subsequence of P consisting of the characters with ranks between i and j
  - A prefix of P is a substring of the type $P[0..i]$
  - A suffix of P is a substring of the type $P[i..m-1]$
- Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- Applications:
  - Text editors
  - Search engines
  - Biological research

Brute-Force Pattern Matching



- The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T, until either
  - a match is found, or
  - all placements of the pattern have been tried
- Brute-force pattern matching runs in time $O(nm)$
- Example of worst case:
  - $T = aaa \ldots ah$
  - $P = aaah$
  - may occur in images and DNA sequences
  - unlikely in English text

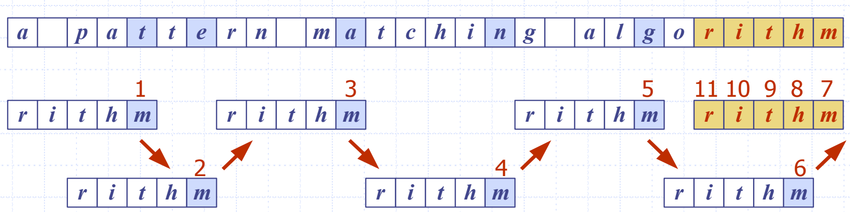
Algorithm *BruteForceMatch*(T, P)
  Input text T of size n and pattern P of size m
  Output starting index of a substring of T equal to P or -1 if no such substring exists
  for $i \leftarrow 0$ **to** $n - m$
    { test shift i of the pattern }
    $j \leftarrow 0$
    while $j < m \wedge T[i + j] = P[j]$
      $j \leftarrow j + 1$
    if $j = m$
      return i {match at i}
    else
      break while loop {mismatch}
  return -1 {no match anywhere}

Boyer-Moore Heuristics

- The Boyer-Moore's pattern matching algorithm is based on two heuristics

  Looking-glass heuristic: Compare P with a subsequence of T moving backwards

  Character-jump heuristic: When a mismatch occurs at $T[i] = c$
  - If P contains c, shift P to align the last occurrence of c in P with $T[i]$
  - Else, shift P to align $P[0]$ with $T[i + 1]$
- Example



Last-Occurrence Function

- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet Σ to build the last-occurrence function L mapping Σ to integers, where $L(c)$ is defined as
  - the largest index i such that $P[i] = c$ or
  - -1 if no such index exists
- Example:
  - $\Sigma = \{a, b, c, d\}$
  - $P = abacab$

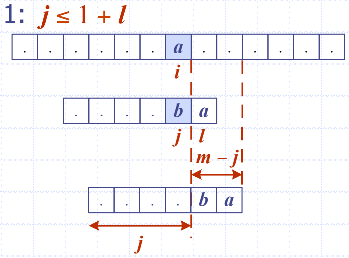
c	a	b	c	d
$L(c)$	4	5	3	-1

- The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- The last-occurrence function can be computed in time $O(m + s)$, where m is the size of P and s is the size of Σ

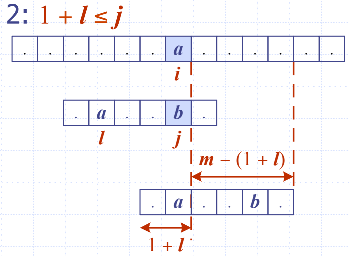
The Boyer-Moore Algorithm

```
Algorithm BoyerMooreMatch(T, P, Σ)
    L ← lastOccurenceFunction(P, Σ )
    i ← m − 1
    j ← m − 1
    repeat
        if T[i] = P[j]
            if j = 0
                return i { match at i }
            else
                i ← i − 1
                j ← j − 1
        else
            { character-jump }
            l ← L[T[i]]
            i ← i + m − min(j, 1 + l)
            j ← m − 1
    until i > n − 1
    return −1 { no match }
```

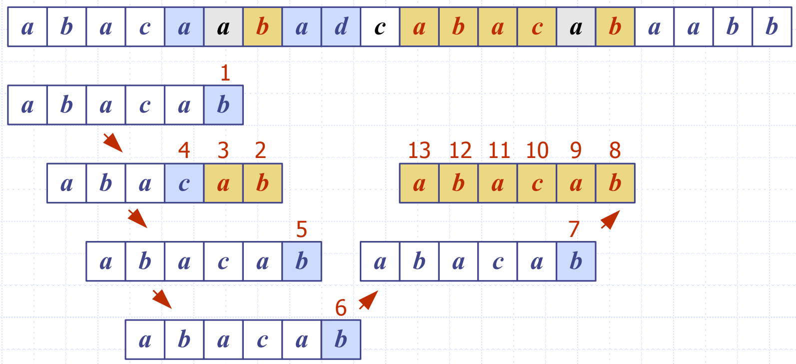
Case 1: $j \leq 1 + l$



Case 2: $1 + l \leq j$

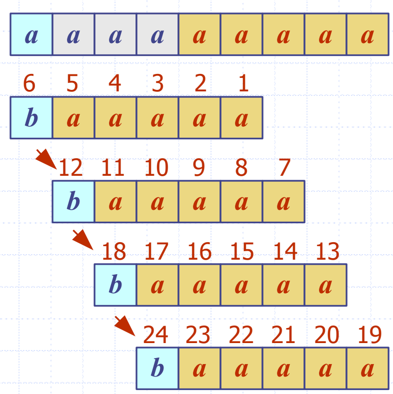


Example



Analysis

- Boyer-Moore's algorithm runs in time $O(nm + s)$
- Example of worst case:
  - $T = aaa \ldots a$
  - $P = baaa$
- The worst case may occur in images and DNA sequences but is unlikely in English text
- Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text

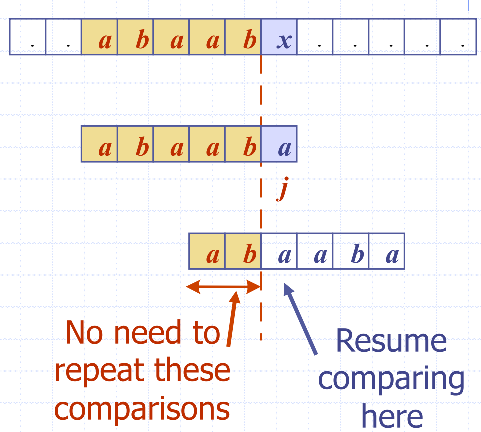


Java Implementation

```
/** Returns the lowest index at which substring pattern begins in text (or else −1).*/
public static int findBoyerMoore(char[ ] text, char[ ] pattern) {
  int n = text.length;
  int m = pattern.length;
  if (m == 0) return 0;                                // trivial search for empty string
  Map<Character,Integer> last = new HashMap<>( );   // the 'last' map
  for (int i=0; i < n; i++)
    last.put(text[i], −1);                       // set −1 as default for all text characters
  for (int k=0; k < m; k++)
    last.put(pattern[k], k);                     // rightmost occurrence in pattern is last
  // start with the end of the pattern aligned at index m−1 of the text
  int i = m−1;                                         // an index into the text
  int k = m−1;                                         // an index into the pattern
  while (i < n) {
    if (text[i] == pattern[k]) {                       // a matching character
      if (k == 0) return i;                            // entire pattern has been found
      i−−;                                             // otherwise, examine previous
      k−−;                                             // characters of text/pattern
    } else {
      i += m − Math.min(k, 1 + last.get(text[i])); // case analysis for jump step
      k = m − 1;                                       // restart at end of pattern
    }
  }
  return −1;                                           // pattern was never found
}
```


The KMP Algorithm

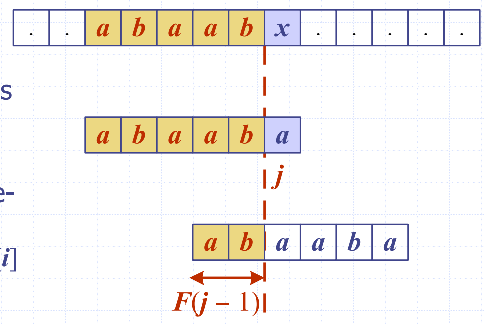
- Knuth-Morris-Pratt’s algorithm compares the pattern to the text in left-to-right, but shifts the pattern more intelligently than the brute-force algorithm.
- When a mismatch occurs, what is the **most** we can shift the pattern so as to avoid redundant comparisons?
- Answer: the largest prefix of $P[0..j]$ that is a suffix of $P[1..j]$



KMP Failure Function

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The failure function $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j-1)$

j	0	1	2	3	4	5
$P[j]$	a	b	a	a	b	a
$F(j)$	0	0	1	1	2	3

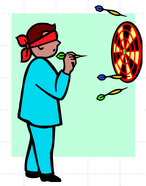


The KMP Algorithm

- The failure function can be represented by an array and can be computed in $O(m)$ time
- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2n$ iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time $O(m + n)$

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j − 1]
            else
                i ← i + 1
    return −1 { no match }
```

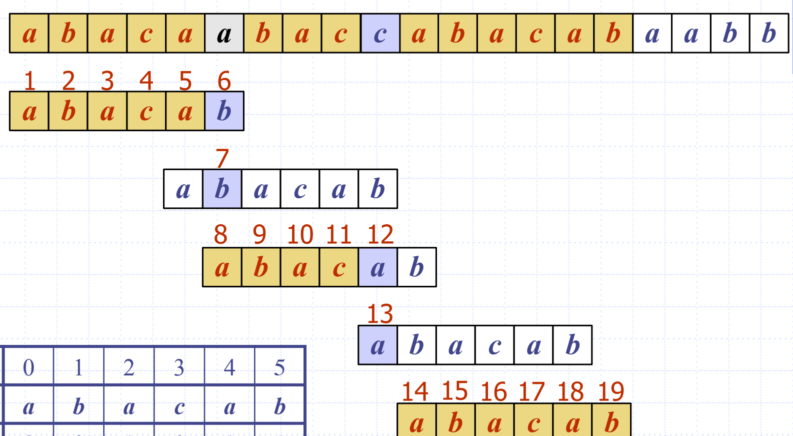
Computing the Failure Function



- The failure function can be represented by an array and can be computed in $O(m)$ time
- The construction is similar to the KMP algorithm itself
- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j - 1) < j$)
- Hence, there are no more than $2m$ iterations of the while-loop

```
Algorithm failureFunction(P)
  F[0] ← 0
  i ← 1
  j ← 0
  while i < m
    if P[i] = P[j]
      {we have matched j + 1 chars}
      F[i] ← j + 1
      i ← i + 1
      j ← j + 1
    else if j > 0 then
      {use failure function to shift P}
      j ← F[j − 1]
    else
      F[i] ← 0 { no match }
      i ← i + 1
```


Example



j	0	1	2	3	4	5
$P[j]$	a	b	a	c	a	b
$F(j)$	0	0	1	0	1	2

Java Implementation

```
/** Returns the lowest index at which substring pattern begins in text (or else −1).*/
public static int findKMP(char[ ] text, char[ ] pattern) {
  int n = text.length;
  int m = pattern.length;
  if (m == 0) return 0;                              // trivial search for empty string
  int[ ] fail = computeFailKMP(pattern);             // computed by private utility
  int j = 0;                                         // index into text
  int k = 0;                                         // index into pattern
  while (j < n) {
    if (text[j] == pattern[k]) {                     // pattern[0..k] matched thus far
      if (k == m − 1) return j − m + 1;              // match is complete
      j++;                                           // otherwise, try to extend match
      k++;
    } else if (k > 0)
      k = fail[k−1];                                 // reuse suffix of P[0..k-1]
    else
      j++;
  }
  return −1;                                         // reached end without match
}
```


Java Implementation, 2

```
private static int[ ] computeFailKMP(char[ ] pattern) {
  int m = pattern.length;
  int[ ] fail = new int[m];                 // by default, all overlaps are zero
  int j = 1;
  int k = 0;
  while (j < m) {                           // compute fail[j] during this pass, if nonzero
    if (pattern[j] == pattern[k]) {         // k + 1 characters match thus far
      fail[j] = k + 1;
      j++;
      k++;
    } else if (k > 0)                       // k follows a matching prefix
      k = fail[k−1];
    else                                    // no match found starting at j
      j++;
  }
  return fail;
}
```