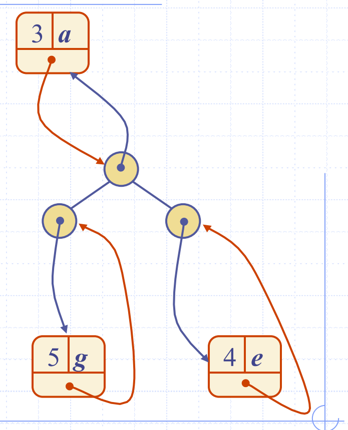
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

Adaptable Priority Queues



Entry and Priority Queue ADTs

- An entry stores a (key, value) pair
- Entry ADT methods:
  - getKey(): returns the key associated with this entry
  - getValue(): returns the value paired with the key associated with this entry
- Priority Queue ADT:
  - insert(k, x) inserts an entry with key k and value x
  - removeMin() removes and returns the entry with smallest key
  - min() returns, but does not remove, an entry with smallest key
  - size(), isEmpty()

Example

- Online trading system where orders to purchase and sell a stock are stored in two priority queues (one for sell orders and one for buy orders) as (p,s) entries:
  - The key, p, of an order is the price
  - The value, s, for an entry is the number of shares
  - A buy order (p,s) is executed when a sell order (p’,s’) with price p’ $\le$p is added (the execution is complete if s’ $\ge$s)
  - A sell order (p,s) is executed when a buy order (p’,s’) with price p’ $\ge$p is added (the execution is complete if s’ $\ge$s)
- What if someone wishes to cancel their order before it executes?
- What if someone wishes to update the price or number of shares for their order?

Methods of the Adaptable Priority Queue ADT

- remove(e): Remove from P and return entry e.
- replaceKey(e,k): Replace with k and return the key of entry e of P; an error condition occurs if k is invalid (that is, k cannot be compared with other keys).
- replaceValue(e,v): Replace with v and return the value of entry e of P.

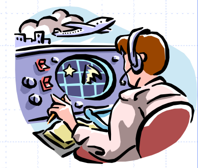
Example

Operation	*Output*	*P*
insert($5,A$)	e_1	$(5,A)$
insert($3,B$)	e_2	$(3,B),(5,A)$
insert($7,C$)	e_3	$(3,B),(5,A),(7,C)$
min()	e_2	$(3,B),(5,A),(7,C)$
key(e_2)	3	$(3,B),(5,A),(7,C)$
remove(e_1)	e_1	$(3,B),(7,C)$
replaceKey($e_2,9$)	3	$(7,C),(9,B)$
replaceValue(e_3,D)	C	$(7,D),(9,B)$
remove(e_2)	e_2	$(7,D)$

Locating Entries

- In order to implement the operations remove(e), replaceKey(e,k), and replaceValue(e,v), we need fast ways of locating an entry e in a priority queue.
- We can always just search the entire data structure to find an entry e, but there are better ways for locating entries.

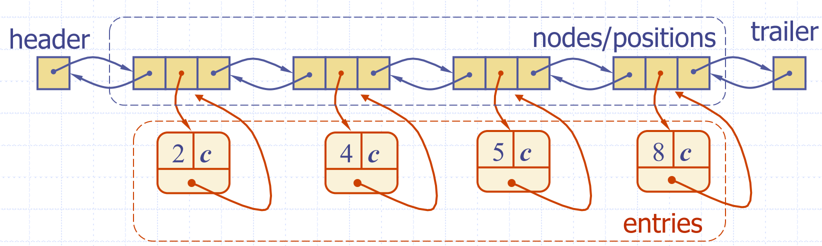
Location-Aware Entries



- A location-aware entry identifies and tracks the location of its (key, value) object within a data structure
- Intuitive notion:
  - Coat claim check
  - Valet claim ticket
  - Reservation number
- Main idea:
  - Since entries are created and returned from the data structure itself, it can return location-aware entries, thereby making future updates easier

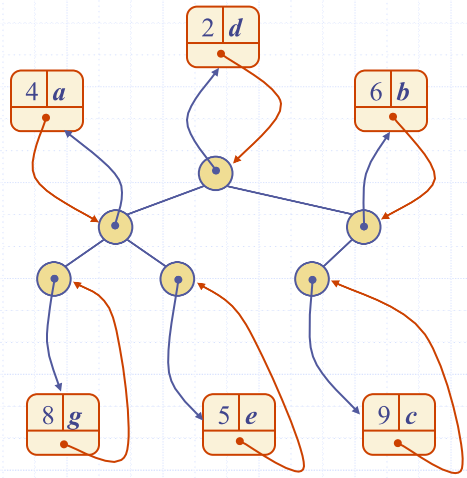
List Implementation

- A location-aware list entry is an object storing
  - key
  - value
  - position (or rank) of the item in the list
- In turn, the position (or array cell) stores the entry
- Back pointers (or ranks) are updated during swaps



Heap Implementation

- A location-aware heap entry is an object storing
  - key
  - value
  - position of the entry in the underlying heap
- In turn, each heap position stores an entry
- Back pointers are updated during entry swaps



Performance

- Improved times thanks to location-aware entries are highlighted in red

Method	Unsorted List	Sorted List	Heap
size, isEmpty	$O(1)$	$O(1)$	$O(1)$
insert	$O(1)$	$O(n)$	$O(\log n)$
min	$O(n)$	$O(1)$	$O(1)$
removeMin	$O(n)$	$O(1)$	$O(\log n)$
remove	$O(1)$	$O(1)$	$O(\log n)$
replaceKey	$O(1)$	$O(n)$	$O(\log n)$
replaceValue	$O(1)$	$O(1)$	$O(1)$

Java Implementation

```java
/** An implementation of an adaptable priority queue using an array-based heap. */
public class HeapAdaptablePriorityQueue<K,V> extends HeapPriorityQueue<K,V>
                                implements AdaptablePriorityQueue<K,V> {

  //---------------- nested AdaptablePQEntry class ----------------
  /** Extension of the PQEntry to include location information. */
  protected static class AdaptablePQEntry<K,V> extends PQEntry<K,V> {
    private int index;                      // entry's current index within the heap
    public AdaptablePQEntry(K key, V value, int j) {
      super(key, value);                    // this sets the key and value
      index = j;                            // this sets the new field
    }
    public int getIndex() { return index; }
    public void setIndex(int j) { index = j; }
  } //----------- end of nested AdaptablePQEntry class -----------

  /** Creates an empty adaptable priority queue using natural ordering of keys. */
  public HeapAdaptablePriorityQueue() { super(); }
  /** Creates an empty adaptable priority queue using the given comparator. */
  public HeapAdaptablePriorityQueue(Comparator<K> comp) { super(comp);}

```


Java Implementation, 2

```java
  // protected utilites
  /** Validates an entry to ensure it is location-aware. */
  protected AdaptablePQEntry<K,V> validate(Entry<K,V> entry)
                                  throws IllegalArgumentException {
    if (!(entry instanceof AdaptablePQEntry))
      throw new IllegalArgumentException("Invalid entry");
    AdaptablePQEntry<K,V> locator = (AdaptablePQEntry<K,V>) entry;   // safe
    int j = locator.getIndex();
    if (j >= heap.size() || heap.get(j) != locator)
      throw new IllegalArgumentException("Invalid entry");
    return locator;
  }

  /** Exchanges the entries at indices i and j of the array list. */
  protected void swap(int i, int j) {
    super.swap(i,j);                                          // perform the swap
    ((AdaptablePQEntry<K,V>) heap.get(i)).setIndex(i);        // reset entry's index
    ((AdaptablePQEntry<K,V>) heap.get(j)).setIndex(j);        // reset entry's index
  }
```

Java Implementation, 3

```java
/** Restores the heap property by moving the entry at index j upward/downward.*/
protected void bubble(int j) {
  if (j > 0 && compare(heap.get(j), heap.get(parent(j))) < 0)
    upheap(j);
  else
    downheap(j);                    // although it might not need to move
}

/** Inserts a key-value pair and returns the entry created. */
public Entry<K,V> insert(K key, V value) throws IllegalArgumentException {
  checkKey(key);                    // might throw an exception
  Entry<K,V> newest = new AdaptablePQEntry<>(key, value, heap.size());
  heap.add(newest);                 // add to the end of the list
  upheap(heap.size() - 1);          // upheap newly added entry
  return newest;
}
```


Java Implementation, 4

```java
/** Removes the given entry from the priority queue. */
public void remove(Entry<K,V> entry) throws IllegalArgumentException {
  AdaptablePQEntry<K,V> locator = validate(entry);
  int j = locator.getIndex();
  if (j == heap.size() - 1)         // entry is at last position
    heap.remove(heap.size() - 1);   // so just remove it
  else {
    swap(j, heap.size() - 1);       // swap entry to last position
    heap.remove(heap.size() - 1);   // then remove it
    bubble(j);                      // and fix entry displaced by the swap
  }
}

/** Replaces the key of an entry. */
public void replaceKey(Entry<K,V> entry, K key)
                          throws IllegalArgumentException {
  AdaptablePQEntry<K,V> locator = validate(entry);
  checkKey(key);                    // might throw an exception
  locator.setKey(key);              // method inherited from PQEntry
  bubble(locator.getIndex());       // with new key, may need to move entry
}

/** Replaces the value of an entry. */
public void replaceValue(Entry<K,V> entry, V value)
                          throws IllegalArgumentException {
  AdaptablePQEntry<K,V> locator = validate(entry);
  locator.setValue(value);          // method inherited from PQEntry
}
```

