

$$f \sim O(g) : \exists n_1 \in \mathbb{N}, \exists c_1 \in \mathbb{R} :$$

$$\forall n > n_1 : f(n) \leq c_1 g(n)$$

---

$$g \sim O(h) : \exists n_2 \in \mathbb{N}, \exists c_2 \in \mathbb{R} :$$

$$\forall n > n_2 : g(n) \leq c_2 h(n)$$

---

$$n_3 := \max(n_1, n_2)$$

$$\forall n > n_3 : f(n) \leq c_1 g(n) \leq c_1 c_2 h(n)$$

$$c_3 := c_1 \cdot c_2$$

$$\exists n_3 \in \mathbb{N}, \exists c_3 \in \mathbb{R} :$$

$$\forall n > n_3 : f(n) \leq c_3 h(n)$$

$$f \sim O(h) \quad \blacksquare$$